**Investments Problem Sheet 7 Lent Term 2024**

**True-false**

1. If interest rates rise, bond prices fall.
2. The value of a portfolio that has zero duration is unaffected by small interest rate changes.
3. If a bond is trading above par (i.e. its market value exceeds its face value) then its interest or running yield is higher than its yield to maturity.
4. The price of a five year bond with a 3% coupon should be exactly half way between the price of two other five year bonds, one with zero coupon, and one with and a 6% coupon.
5. In general, you get a higher rate of return from long-dated bonds than from short-dated bonds.
6. If forward rates are higher than spot rates, yields on longer-dated bonds are higher than yields on shorter-dated bonds.
7. The dirty price of a bond tends to be more variable than the clean price.

**Problems**

1. The table below lists the prices of zero-coupon bonds of various maturities, expressed as a price per $1000 of principal. Calculate the yields to maturity of each bond and the sequence of implied forward rates.

|  |  |
| --- | --- |
| Maturity (years) | Price of bond ($) |
| 1 | 943.40 |
| 2 | 898.47 |
| 3 | 847.62 |
| 4 | 792.16 |

Assuming that the *pure expectations hypothesis* is valid, calculate the expected price path of the four-year bond as time passes. What is the rate of return of the bond in each of the four years?

2. The yield-to-maturity on one-year and two-year zero-coupon bonds is currently 7% and 8%, respectively. The Treasury plans to issue a two-year bond that pays an annual coupon of 9% of face value. The face value of the bond is £100.

a. At what price will the bond sell?

b. What will be the yield-to-maturity of the bond?

c. If the *pure expectations hypothesis* for the term structure of interest rates is correct, what is the market expectation of the price that the bond will sell for next year?

d. Repeat the calculation in (c) assuming the *liquidity preference hypothesis* holds with a liquidity premium of 1%.

3. Without doing any detailed computations, and assuming that the term structure is flat at 5%, put the following bonds in order of increasing duration:

(a) a perpetual 5% coupon bond

(b) a perpetual 3% coupon bond

(c) a 25 year zero coupon bond

(d) a 10 year floating rate bond that pays LIBOR

(e) a 5 year 5% coupon bond

(f) a 5 year 6% coupon bond

4.The YTM on 1-year zero-coupon bonds is 5% and the YTM on 2-year zero-coupon is 6%. The YTM on 2-year coupon bonds with coupon rates of 12% (paid annually) is 5.8%. What arbitrage opportunity is available for an investment banking firm? What is the profit on the activity?